Gravitational Lensing

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Abstract Gravitational lensing, one of the outcomes of Einstein's theory of special relativity, offers insights into the universe. This paper explores gravitational lensing comprehensively; covering its theoretical foundations, observable evidence, applications, and future directions and challenges.

Beginning with the theoretical framework, we explain how massive objects warp spacetime, bending light rays and creating observable distortions in background sources' images. We discuss notable sections in the gravitational lensing field and key equations.

Observations are crucial for detecting and studying lensing phenomena. We discuss times we have used lensing, how we are currently using it, and where it can go from here.

Looking ahead, many new instruments are being formed to help push the boundaries of space exploration. Telescopes like the JWST and the Large Synoptic Survey Telescope hold hope for advancing these studies.

1 Introduction

In 1912, Einstein first hypothesized the idea of light warping around a heavy mass object for an observer, like a lens, through the use of special relativity [Ein12]. This idea was then observationally confirmed in 1919 during a solar eclipse. Scientists immediately jumped on theories of what gravitational lensing could do, such as using it to map out distant galaxies, find distant and faint objects, or even see the effects of dark matter.

By definition, light rays are deflected as they pass through a nonuniform gravitational field. Newton's theory of gravitation was able to calculate the deflection of light, assuming it behaved as a particle; when General Relativity was introduced, the effect of light deflection was doubled through calculations [BS01].

One of Einstein's many hypotheses was the idea of lens configuration in 1936: when the source and lens are perfectly aligned, the observer should see a ring of the lens around the source. These rings are now known as the "Einstein-ring" or the "Chwolson-ring", Chwolson being the first to publish his hypothesis regarding the rings in 1924 [Wam98].

Figure 1: A luminous red galaxy has distorted the light from a more distant blue galaxy. The result shows a horseshoe-like ring around the original red galaxy – Image credit: NASA, ESA

Gravitational lensing can often result in a multiple-image effect as seen in figure 1 above. When the light follows many different paths caused by gravity, the observer will see more than one image of the same object. This effect is incredibly helpful for scientists because it allows a better look at different aspects of the object they are trying to view.

Although gravitational lensing was confirmed and observed in 1919 with a photograph published by Arthur Eddington, it took many years to understand how to use it experimentally with other outside sources. And until, around 40 years ago, there was no branch in astrophysics to study its uses.

2 Theoretical Foundations

Light follows the straightest path available and sometimes that path must curve. Gravity warps spacetime and thus affects the paths that light travels through, creating effects such as gravitational lensing. The larger the mass, the larger the gravity, and the larger the bending of light.

Einstein was able to create an equation relating the light deflection angle to mass and distance after completing his theory of relativity.

$$\tilde{\alpha} = \frac{4GM}{c^2}\frac{1}{r} \tag{1}$$

In this, $\tilde{\alpha}$ is the reflection angle, G is the gravitational constant, c is the speed of light, M is the mass, and r is the distance traveled. When plugging in solar values, we get 1.74 arcsecs. This value is double the value found with Newtonian gravity which is 0.83 arcsecs.

2.1 Thin Screen Approximation

Assume that the relative velocities of the lens, source, observer, and Newtonian potential are much smaller than the speed of light. From this, we can say that deflection takes place at only one distance.

Assuming from this that the action of lensing is created by a slight area of inhomogeneity between the source and the observer.

For this approximation, we use the Friedmann-Robertson-Walker metric for spacetime.

$$ds^2 = (1 + \frac{2\Phi}{c^2})c^2 dt^2 - a^2(t)(1 - \frac{2\Phi}{c^2})d\sigma^2 \tag{2}$$

Adjusting equation (1) to fit this approximation (and assuming circular symmetry), we get

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2\xi} \tag{3}$$

($\hat{\alpha}$ is the same variable as $\tilde{\alpha}$ in this instance) where ξ is the angular position of the source or image as we can see in figure 2 below [Pri20].

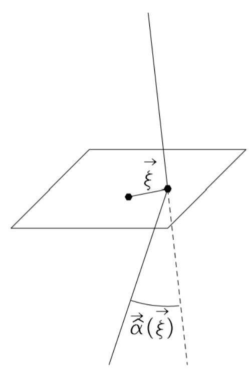

Figure 2: Thin screen approximation figure.

2.2 The Lens Equation

For this section, we will use a simple gravitational lens scenario which only involves a point source (S), a point lens (L), and an observer (O). This scenario will always give at least two images of the source; the shear is the variable that is responsible for creating more images. These images are tangents of the real light paths. This can all be seen in better detail through figure 3 below.

Another important figure (4) shows the relation between different angles and distances used in lensing that can be seen below.

From this figure, we can find many equations:

$$\theta D_S = \beta D_S + \tilde{\alpha} D_{LS} \tag{4}$$

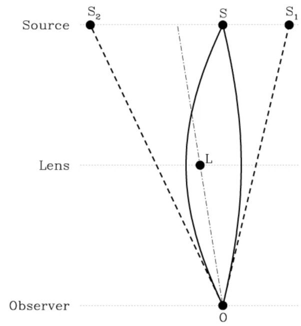

Figure 3: Simple gravitational lens scenario: The lens L produces two images, S_1 and S_2, of the source, [Wam98]

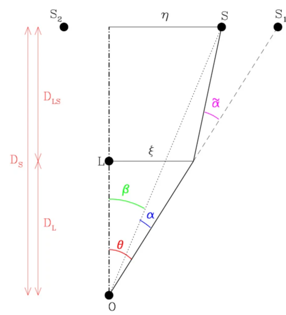

Figure 4: Lens equation relating angles and distances for $/Tildealpha << 1$, [Wam98]

(for θ, β, $\tilde{\alpha} << 1$ which is almost always fulfilled in relevant scenarios)

$$\beta = \theta - \alpha(\theta) \tag{5}$$

(where $\alpha(\theta) = \frac{D_{LS}}{D_S}\tilde{\alpha}(\theta)$). These all assume symmetric mass distribution; however, if we adjust equation 5, we get a vector-valued lens equation that can be used for non-symmetric mass cases.

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) \tag{6}$$

2.3 Microlensing

Microlensing is currently the only way to detect planets at an incredibly far distance from Earth. It is different from the usual lensing method because it focuses on much smaller objects, like single stars and exoplanets, compared to larger objects, like galaxies or galaxy clusters.

Astronomers use microlensing primarily for discovering exoplanets that are at a great distance from us, that are very far away from the star they orbit, or free-floating exoplanets. In this scenario, the exoplanet acts as the lens that brightens the source.

The downside to microlensing is that after the initial discovery of the exoplanet, we are unlikely to be able to find the planet again due to it being so small and likely a large orbit around its star, if an orbit at all. This method is also, currently, only able to detect one exoplanet at once [Soc20].

Despite the setbacks, microlensing is still one of the ways astronomers discover exoplanets. Though it isn't quite as common as finding them through transit or radial velocity discoveries, it has many more than its rivals as we can see below in figure 5.

Confirmed Exoplanet Statistics

Discovery Method	Number of Planets
Astrometry	3
Imaging	68
Radial Velocity	1087
Transit	4166
Transit timing variations	29
Eclipse timing variations	17
Microlensing	210
Pulsar timing variations	7
Pulsation timing variations	2
Orbital brightness modulations	9
Disk Kinematics	1

Figure 5: Exoplanet discovery statistics provided by NASA [NAS24]

3 Weak Gravitational Lensing: A Closer Look

I wanted to take a deeper look into a particular section of gravitational lensing which happens to be "weak gravitational lensing". Weak gravitational lensing is exactly how it sounds, it refers to the slight distortion that only shows small changes to the shapes or orientations of the background objects. Weak lensing affects the light brightness as well, allowing us to see dimmer stars in a brighter state. These changes can only be quantified through statistical measures such as shear and convergence fields. Shear is the stretch or compression, convergence fields are the magnification or demagnification of the observed object.

The shear and convergence fields due to weak gravitational lensing can be used to determine the matter distribution between us and the sources. Using what we know and can assume with closer objects, we can find the magnification caused by them on background sources. [BS01]

3.1 Shapes, Sizes, and Tranformations

To fully understand the image we are seeing from the lens, we need to determine the makeup of the lens and the source. Everything can have an impact on how we measure the source through the lens, so we need to be sure we aren't measuring the wrong things.

One important equation involves the surface brightness $I(\theta)$ at angular position θ and the central area $\bar{\theta}$ of an observable galaxy.

$$\bar{\theta} = \int d^2\theta q_I(I(\theta)) \int d^2\theta q_I(I(\theta)) \quad (7)$$

$q_I(I)$ is a specific weight function. If $q_I(I) = H(I - I_{th})$ (the Heavyset step function), then $\bar{\theta}$ is the center of the area that is enclosed by a limiting isophote $I = I_{th}$. If $q_I(I) = I$, then $\bar{\theta}$ is the center of light. Lastly, if $q_I(I) = IH(I - I_{th})$, then $\bar{\theta}$ is a mix between the previous two as the center of light within the limiting isophote. So, with y=our varying solutions to $q_I(I)$, we can define the tensor of second brightness moments with Q_{ij}.

$$Q_{ij} = \int d^2\theta q_I(I(\theta))(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j) \int d^2\theta q_I(I(\theta)), i, j \in \{1, 2\} \quad (8)$$

With Q defined, and $q_I(I)$ chosen such that the integrals eventually converge; we can define the size of an image with ω.

$$\omega = (Q_{11}Q_{22} - Q_{12}^2)^{1/2} \quad (9)$$

This will then be proportional to the solid angle that is enclosed by the limiting isophote (if $q(I)$ is the step function).

Now that we have the size, we can find the shape with the complex ellipticity χ.

$$\chi = Q_{11} - Q_{22} + 2iQ_{12}Q_{11} + Q_{22} \quad (10)$$

If the image has elliptical isophotes with an axis ratio that is less than or equal to one, then $\chi = (1 - r^2)(1 + r^2)^{-1}e^{(2i\vartheta)}$. The phase of χ is twice that of the position angle ϑ of the major axis. We do this to ensure that the complex ellipticity is unchanged if the axis is rotated 180°.

To further define the complex ellipticity of the source $\chi^{(s)}$ alongside the second brightness moments $Q_{ij}^{(s)}$ transform, we use the following equation that includes reduced shear g.

$$g(\theta) = \gamma(\theta)1 - \kappa(\theta) \quad (11)$$

$$\chi^{(s)} = \chi - 2g + g^2\chi * 1 + |g|^2 - 2R(g\chi*) \quad (12)$$

In the equation, the asterisk implies a complex conjugation. The equation shows that the transformation of image ellipticities depends purely on the reduced shear instead of the separated version (shear and surface mass density). Because of this, the reduced shear of functions are the only values accessible through measuring image ellipticities [BS01].

When using different ellipticity parameters, we exchange χ with ε. They can be related to each other through the following equation:

$$\varepsilon = \chi 1 + (1 - |\chi|^2)^{1/2}, \chi = 2\varepsilon 1 + |\varepsilon|^2 \tag{13}$$

3.2 Magnification Effects

Along with the distortion of image shapes, gravitational light can also magnify the image which makes the surface brightness invariant. This magnification not only adjusts the size but also changes the flux of the images. Two of the main strategies used to measure magnification are through the change in local source counts, or through the change of image sizes at a fixed surface brightness.

The redshift distribution is adjusted with the flux through the following equation:

$$p(z; > S, \kappa, \gamma) = n_0[> \mu^{-1}(z)S, z]\mu(z)'\mu{-}1(z')n_0[> \mu^{-1}(z')S, z'] \tag{14}$$

Where S is a fixed flux, p is the redshift distribution, z is the redshift, μ is the magnification, and $n_0(> S, z)dz$ is the unlensed number density of galaxies with redshift between dz and z and a flux larger than S. In general, redshift information is hard to obtain spectroscopically, we can only observe the redshift-integrated counts through this equation:

$$n(> S) = \int dz 1\mu(z)n_0(> \mu^{-1}(z)S, z) \tag{15}$$

We can calculate the unlensed counts as the coming equation. We use the equation since many different fluxes need to be taken into account.

$$n_0(> S, z) = aS^{-\alpha}p_0(z : S) \tag{16}$$

The α variable depends on the wave band of the observation and $p_0(z : S)$ is the redshift probability distribution of galaxies with flux ¿S. The equation can be altered by combining the last two equations to find both lensed and unlensed source counts.

$$n(> S)n_0(> S) = \int dz\mu^{\alpha-1}(z)p_0(z : \mu^{-1}(z)S) \tag{17}$$

The next thing I want to talk about is the size effect of magnification. We can use I as some measurement of the surface brightness, ω is the solid angle of an image, and the redshift z. We can denote $n(, I, z)d\omega$ as the number density of images within $d\omega$ of ω. We can then see the relation between the lensed and unlensed number density.

$$n(\omega, I, z) = 1\mu^2 n_0\omega\mu, I, z \tag{18}$$

We can simplify that in the case of a moderately small lens redshift (the magnification can be assumed to be locally constant, regardless of redshift).

$$\langle\omega\rangle(I) = \mu\langle\omega\rangle_0(I) \tag{19}$$

If the mean image size can be measured without lensing, then we can find the local value of the magnification, μ, by comparing the observed image sizes to the images in the blank fields.

Regarding the involvement of shear with magnification, we will be considering a small patch of sky that contains an N number of galaxy images without lensing (the lens parameters κ and γ can be assumed as constant). The signal-to-noise ratio can be obtained through the dispersion of a shear estimate from averaging galactic ellipticities [BS01].

$$SN_{shear} = |\gamma|\sigma_\varepsilon N \tag{20}$$

Where σ_ε is the dispersion of the intrinsic source ellipticity $\varepsilon^{(s)}$.

For the galaxy number counts, the signal-to-noise ratio (assuming Poissonian noise) is

$$SN_{counts} = 2\kappa|\alpha - 1|N. \tag{21}$$

Then, for the magnification estimate, the signal-to-noise ratio is

$$SN_{size} = 2\kappa\sigma(I)N. \tag{22}$$

4 Observable Evidence

As discussed in the introduction, a solar eclipse in 1919 confirmed to astronomers that stars near the Sun slightly bend around it, causing the stars to look out of their natural position. Although this was a large boost in Einstein's theory of relativity, it wasn't until around 70 years later that astronomers could use it to discover a new object: a double quasar named Q0957+561.

The Hubble telescope, launched in April 1990, captured an image with four images of a quasar about five months later. Not even a decade later, Hubble updated the amount of known optical gravitational lenses like the ones talked about above. Scientists were able to update Hubble's survey cameras in 2002 according to these new lensing techniques, the images that returned galaxy clusters that gravitationally lensed incredibly distant objects never seen before.

Hubble has found the two most distant stars ever seen. The first star Icarus was found in 2018, it is 9 billion light-years away. The second star Earendel was found in 2022 and it is approximately

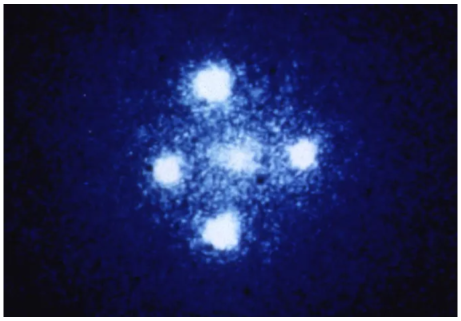

Figure 6: Hubble's Faint Object Camera picture of a multi-imaged quasar, an example of an Einstein Cross. The central galaxy is 400 million light-years away, the quasar is 8 billion light-years away which is about 20 times further – Image credit: NASA, ESA, STScI

12.9 billion light-years away. The Hubble captured its light when it was only 7% of its current age. Icarus was seen at about 30% of its current age. Earendel was able to be seen because it was perfectly positioned on a ripple in spacetime that gives it extreme magnification, this magnification allows it to be seen through its host galaxy and appear to us as a red smear.

As of 2023, Hubble and NASA's James Web Space Telescope partnered to observe a galaxy cluster that is 4.3 billion light-years away. So far, they have identified 14 objects with varying light. 12 of the objects are likely stars or star systems that are briefly magnified to the extent that we were able to see them. The other two are likely background galaxies.

Figure 7: The combined efforts of JWST and Hubble were able to capture the galaxy cluster MACS0416 – Image credit: NASA, CSA

A major use of gravitational lensing revolves around dark matter. We can prove the existence of dark matter through the use of lensing and seeing the effect of light as it passes through. In images, dark matter can be 'seen' where the source images revolve around. Since there is no astronomical object as a lens, we can only assume that dark matter has affected the sources [Bol24].

In 2006, through using microlensing, astronomers were able to find an exoplanet that was only five Earth masses, which was the lowest mass at that time. It was 22,000 light-years away, near the center of our galaxy, making it the farthest planet discovered as well [Soc20].

In the case of weak gravitational lensing, a team by Princeton University, Carnegie Mellon University, and the astronomical communities of Japan and Taiwan set up a sky survey called the Hyper

Suprime-Cam (HSC) out of Japan's 8.2-meter Subaru Telescope in Hawai'i. The HSC collects data from over 25 million galaxies that will help create precise analyses of weak gravitational lensing. The data is mainly used to find out more effects of dark matter on the universe based on visual objects like galaxies and stars [Duf21].

As of last year, they have found that the value of the "clumpiness" of the universe's dark matter is approximately 0.78 (this is compared to the original value of 0.83 based on radiation emitted from the cosmic microwave background). This result hints at a different type of physics beyond the Lambda-Cold Dark Matter cosmological paradigm [Duf21].

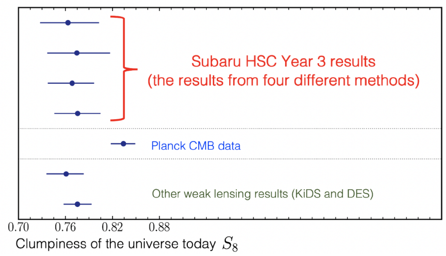

Figure 8: The results of the HSC data towards the "clumpiness" value of dark matter in the universe. Credit: IPMU)

5 Applications

As we have seen, there are many applications of gravitational lensing, and that only grows the more we explore space with different instruments. Gravitational lensing can be used to map out the cosmos. Because of this, we can use lensing to map out large galaxies or dark matter halos.

The evolution of information on dark matter is becoming so important in the astronomy field since it could end up explaining mysteries in the cosmos. Gravitational lensing has the ability and will only get better at finding and mapping out dark matter. It has already given us the idea of structures of dark matter such as a sort of halo around galaxies.

Through strong gravitational lensing phenomena like Einstein rings or multiple images, we can analyze the source in extreme detail. We can then use the information from all the galaxies we observe to create parameters for galaxy formation.

Another thing lensing can be used for is probing stellar populations and chemical compositions of lensing galaxies. This can also set up better foundations for galaxy compositions relating to the ages of the galaxies.

Going back to the team with the HSC: the result of their data may change physics forever if it is correct. We have used the radiation value of the CMB of clumpiness for ages; the fact that it may be incorrect puts our current physics knowledge at risk [Duf21].

6 Future Directions and Challenges

Gravitational lensing is constantly changing and evolving as technology changes and evolves. Hubble and JWST continue to examine and study gravitational lensing as well as several other telescopes like the HSC.

There is no way to know exactly where the future of gravitational lensing will go, but I know that it will become better and sharper over time. This will help us in the long run to better understand several things such as different cosmological models or narrowing down the Hubble constant.

Although there is an unlimited amount of information we can receive from gravitational lensing, there is an enormous amount of hops that we need to jump through to get to that point. One of the main challenges is the technology that is currently available and the uncertainties that they would have. Our current technology also requires very precise calibration which is difficult to achieve or fix in deep space.

Another large challenge is the massive amounts of data that telescopes receive from their observations. The ability to ensure that data can be stored and processed correctly can be quite difficult and will hopefully become easier in the future.

7 Conclusion

In conclusion, gravitational lensing was and continues to be an incredible phenomenon, revealing the relationship between matter and spacetime. Gravitational lensing acts as a sort of cosmic magnifying glass that allows us to peer into the past and better understand the composition of our universe.

From the beginning, in 1912, Einstein predicted gravitational lensing through his theory of special relativity, and since then it has flourished into a magnificent field of study. From the discovery of multi-imaged quasars to dark matter found in clumps around the cosmos, the use of lensing continues to grow and evolve.

This field has been growing exponentially and doesn't seem to be slowing down any time soon which gives me great hope for the future of cosmology. With the use of gravitational lensing, we will eventually be able to create a map of our universe and get a better grasp on how we and all other astronomical objects came to be. Though, like any other field of study, there are challenges; however, these challenges should only push us to want to be better and strive for more.

Ultimately, gravitational lensing has been able to help open our eyes to the beauty and complexity of the universe. With each new observation and discovery, we get closer to unraveling the mysteries of our universe. The cosmos is incredibly vast and essentially never-ending, but gravitational lensing can help us make it a little less daunting to think about.

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